1.1

1.2 Antenna Measurements

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1.3 Introduction

1.3.1 Friis Equation

Traditional Antenna measurement has until recent years required custom techniques and hardware. Measurement of small radiated signals over wide dynamic ranges required the use of individually calibrated detectors and low noise high gain receivers. These systems were limited to measurement at a single frequency making frequency sweeps very time consuming. Performing a measurement sweep over Azimuth and Elevation required control and synchronization through the use of a computer. The rapid and unforeseeable evolution of the computer has given antenna measurement a technological boost. Vector based network analyzers and simple stepper motors are easily controlled by micro computers for large amounts of data collection. Vector based network analyzers also make possible antenna separation measurements based on group delay.

This chapter will present measurement considerations associated with antenna measurements made using fast efficient data collection and processing systems. Far field criteria, reflections and multipath estimates are important knowledge for measurement dynamic range consideration. The choice of Az-EL movement sequencing and its dependence on mechanical considerations and symmetry is presented. Lastly an actual measurement made using a commercially available system is performed using two different techniques one based on use of a calibrated reference and the second based on the 3-antenna method. Both methods are based on the solution of Friis formula:

$$P_r = (\frac{\lambda}{4\pi d})^2 G_t G_r P_t \qquad (1.1)$$

Where:

$$P_r =$$ Received power level

$$P_t =$$ Transmit power level

$$\lambda =$$ Transmit wave length

$$G_t =$$ Gain of the transmit antenna

$$G_r =$$ Gain of the transmit antenna

$$G_r =$$ Gain of the receive antenna

$$d =$$ Separation distance between antennas

It is convenient to express Friis formula in terms of $S21^2 = P_r / P_t$ and dB:

$$S21_{dB} = P_L^{dB} + G_t^{dB} + G_r^{dB} \tag{1.2}$$

Where the path loss is defined as:

$$P_L^{dB} = 20Log(\frac{\lambda}{4\pi d}) \tag{1.3}$$

There are several useful measurement constants that can be derived from this simple equation. First note that at a distance of one wavelength from the source the path loss is:

$$P_L^{dB}(\lambda = d) = 20Log(\frac{1}{4\pi}) = -21.98dB \tag{1.4}$$

The Path Loss equation can then be written as:

$$P_L^{dB} = -21.98 + 20Log(\frac{\lambda}{d}) \tag{1.5}$$

Note that each time the distance increases by a factor of 10 the path loss increases 20dB. Similarly if the distance from the source doubles then the

path loss increases by 6dB. These three relationships are summarized as follows:

1. Minimum path loss between any two antennas is -22dB
2. Path loss increases 6dB for each distance factor of 2 from the source
3. Path loss increases 20dB for each distance factor of 10 from the source

Measurement Estimate

Assume a measurement is to be performed using a calibrated horn and a dipole antenna. The separation distance between the two antennas is to be 200λ . The calibrated horn is known to have a gain of 8dB. Estimate the link loss from the above relationships.

Solution
We know at a distance of one λ the loss is 22dB and increases 20dB for every separation factor of 10 so at 10λ we are at 22dB + 20dB = 42dB and at 100λ $P_L(100\lambda) = 42$dB + 20dB = 62dB. Now at double the distance we encounter an additional 6dB loss resulting in a total path loss of:

$$P_L(200\lambda) = 62 + 6 = 68\text{dB}$$

Next add the gain of the horn plus 2dBi for a typical dipole and the link loss estimate is:

$$S21_{dB} = -68 + 8 + 2 = -58dB$$

1.4 Antenna Measurement Considerations

The measurement of an antenna under test (AUT) requires knowledge of the desired test field wavefront at the plane of the AUT. If one wishes to perform measurements on an AUT using a linear $\overline{E}_y$ test field, then tests must be done to insure $\overline{E}_Y(x,z) \sim 0$ or are acceptably small. Test field energy present in the x,z directions will manifest as a loss and affect the AUT gain. Typically such error components are specified as "not to exceed".

$$\overline{E}_Y$$

$$\overline{E}_Y(r)$$

Linearly (Y) Polarized
Test Antenna

$$\overline{\overline{E}}_X$$

r

AUT
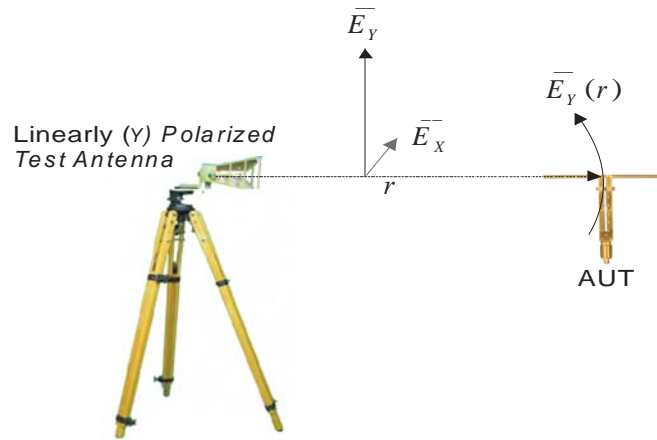
Fig 1-1 In this system test field components are required to be linear and constant along the AUT vertical Y axis

In measurement systems where the AUT axis is not coincident with the mechanical movement structures' axis the AUT will move through the test field in both elevation and separation r. The change in separation will cause a test signal amplitude change.

$$P_L^{dB}(d = r) = 20Log(\frac{\lambda}{4\pi r}) \qquad (1.6)$$

$$P_L^{dB}(d = r + \Delta r) = 20Log(\frac{\lambda}{4\pi(r + \Delta r)}) \qquad (1.7)$$

If we define the percentage of d change as $1 + \dfrac{\Delta r}{r} = \eta$ then the corresponding increase (or decrease) in test signal level (dB) is:

$$\Delta P_L^{dB} = 20Log(\frac{\lambda}{4\pi(r+\Delta r)}) - 20Log(\frac{\lambda}{4\pi r}) = 20Log(\frac{\lambda}{4\pi r \eta}) - 20Log(\frac{\lambda}{4\pi r}) \qquad (1.8)$$

$$\Delta P_L^{dB} = -20Log(\eta) \qquad (1.9)$$

This equation is a simple expression for path loss change in terms of relative distance. The previous useful relationships can easily be checked. For example is the distance between source antenna the AUT doubles then,

$$\eta = 2 \, and \, \Delta P_L^{dB} = -20Log(2) = -6dB$$

An antenna measurement system which shifted the AUT toward the source 1% of the separation distance would change the path loss by,

$$\eta = 1 - .01 = .99 \, and \, \Delta P_L^{dB} = -20Log(.99) = .087dB$$
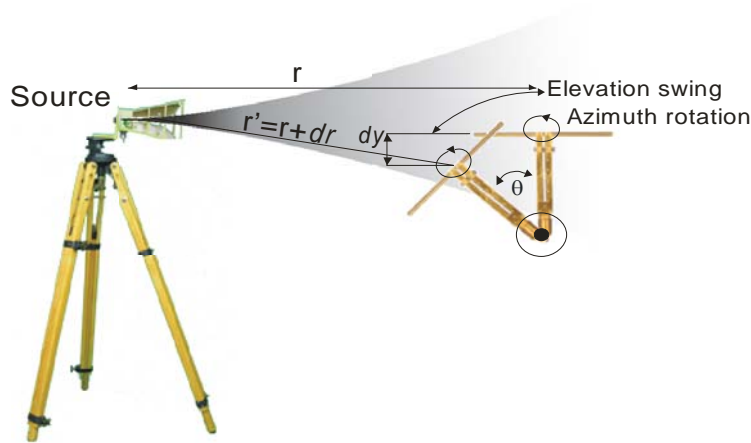
Fig 1-2 AUT test setup showing the movement (*d*x,*d*y) of the AUT phase center through the radiated test field

When performing a measurement the uncertainty of path length can significantly affect accuracy. As the path length decreases toward a single wavelength the gradient increases. Path loss is defined as:

$$P_L = (\frac{\lambda}{4\pi r})^2$$

For a given range configuration one must consider the sensitivity to path length inaccuracies. If the range were 36in and the test frequency were 1 to 6GHz then what is the error/in? The path loss slope is:

$$\frac{\partial P_L}{\partial r} = -2(\frac{\lambda^2}{(4\pi)^2 r^3}) = -37.86\mu \text{ at 36in, 1GHz}$$

$$= -1.052\mu \text{ at 36in, 6GHz}$$

Or over frequency:
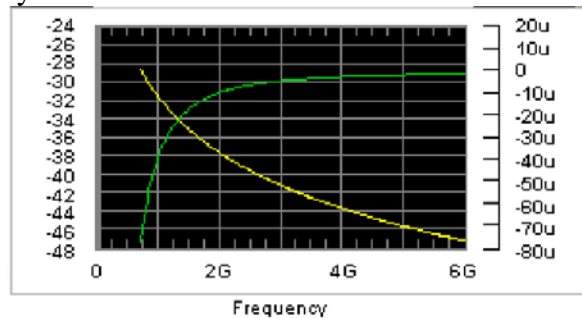
Path Loss(dB) (yellow,left) and Path sensitivity (green,right)

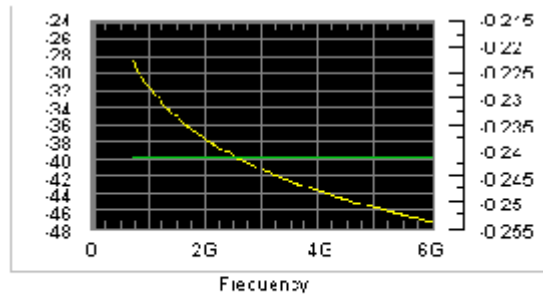A useful sensitivity figure of merit can be derived from the dB path loss slope. Let the db path loss be defined as:

$$P_L^{dB}(r) = 20Log(\frac{\lambda}{4\pi r})$$

The path slope of the db path loss is:

$$\frac{\partial P_L^{dB}(r)}{\partial r} = \frac{20Log(e)}{r} db/in \qquad\qquad 1.95$$

This means the db sensitivity is constant and independent of frequency. If the same plot is performed for 36in and 700MHz to 6GHz we have:

Path Loss(dB) (yellow,left) and Path sensitivity
(dB/in) (green,right) showing the frequency
independence.

Equation 1.95 is an important and useful parameter.

In a typical AUT test setup it is necessary to move either the source or the AUT while making measurements. Fig 1-3 shows and exaggerated elevation movement of the AUT. The dy movement represents the new vertical position of the AUT in the test field while the dr movement represents the inverse square variation. The two variations give rise to a test field variation $dE(r,y)$ at the AUT phase center. The dr variation caused by inverse square(from the source) is simply $\Delta P_L^{dB} = 20 Log(\eta)$ while the dy variation is due to the physical properties of the test field antenna and must be measured. Diagrammatically the AUT movement looks like:
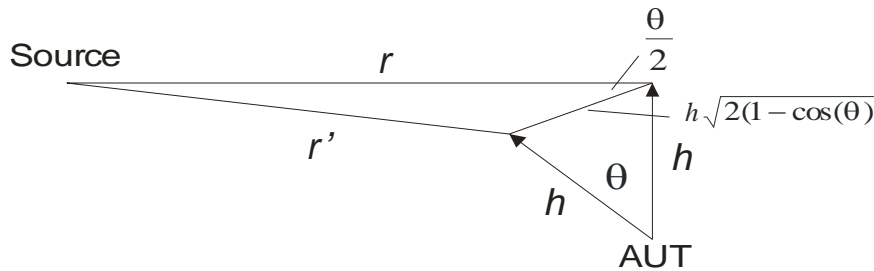
Fig 1-3 The application of the Law of Cosines yields the following simple equation for the source path variation

$$r'^2 = r^2 + 2h^2(1 - \cos(\theta)) - 2rh\sqrt{2(1 - \cos(\theta))}\cos(\frac{\theta}{2}) \qquad (1.10)$$

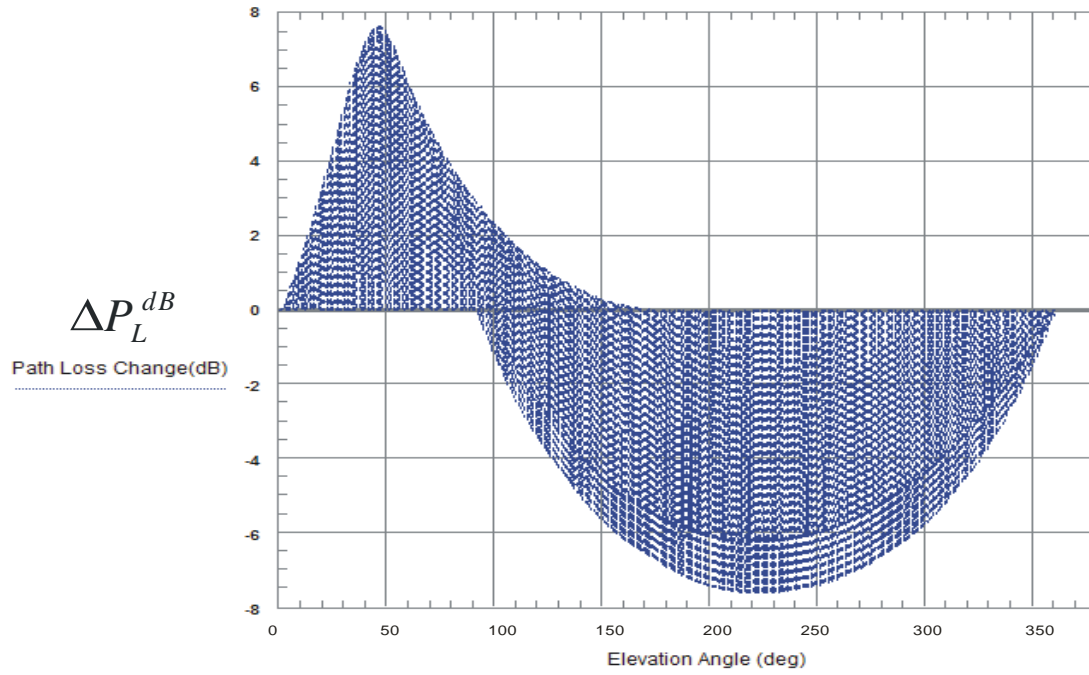And $\eta^2 = (\frac{r'}{r})^2 = 1 + 2\frac{h}{r}[\frac{h}{r}(1 - \cos(\theta)) - \sqrt{2(1 - \cos(\theta))}\cos(\frac{\theta}{2})] \qquad (1.11)$

The dB correction factor is then:

$$\Delta P_L^{dB} = 20Log(\eta) = 10Log[1 + 2\frac{h}{r}[\frac{h}{r}(1 - \cos(\theta)) - \sqrt{2(1 - \cos(\theta))}\cos(\frac{\theta}{2})]] \qquad (1.12)$$

h/r = .001 to 1

$$\Delta P_L^{dB}$$

Path Loss Change(dB)

Fig 1-4 Cumulative plot of path loss dB change for a wide range of *h/r*

Test Field Measurement – Boresite method
The test field variation $E_y(r,y)$ from the source antenna must be measured with an isotropic probe antenna *Gr~1*. It is necessary to align the phase centers of source and the AUT. This can be done using a "boresight" alignment tool. The isotropic antenna is moved ± y vertical distance to determine the test field variation.
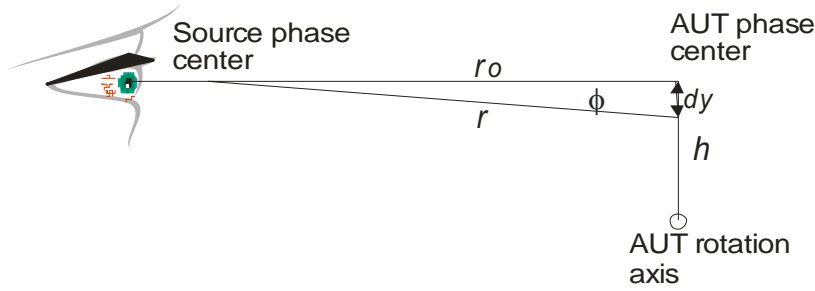
Fig 1-5 Boresite alignment of the phase centers and probing the test field for variations.

The elevation rotation θ shown in Fig 1-5 suggests $dy<0$ such that the test probe movement should be in the –y direction.
With the y origin at the AUT phase center the dB change in source gain is:

$$\frac{\partial G_t^{dB}(y)}{\partial y} = \frac{\partial S21(y)_{dB}}{\partial y} - \frac{\partial 20Log(\frac{\lambda}{4\pi r})}{\partial y} \tag{X.13}$$

$$= \frac{\partial S21(y)_{dB}}{\partial y} + 20\frac{\partial Log(r)}{\partial y} \tag{X.14}$$

Applying the Log identity $\dfrac{d(Log(u))}{dx} = \dfrac{Log(e)}{u}\dfrac{du}{dx}$

$$u = r = \sqrt{r_o^2 + (y)^2} \quad so \quad \frac{20\partial Log(r)}{\partial y} = \frac{20Log(e)}{(r_o^2 + y^2)}y \tag{X.15}$$

The dB link equation is written $S21_{dB} = P_L^{dB} + G_t^{dB} + 1$ and the test antenna gain along the boresight is:

$$G_t^{dB}\Big|_{y=0} = S21_{dB}\Big|_{r=r_0} -1 - P_L^{dB} \qquad \text{(X.16)}$$

The dB change in signal strength with vertical movement of the isotropic probe antenna is:

$$\frac{\partial G_t^{dB}(y)}{\partial y} = \frac{\partial S21(y)_{dB}}{\partial y} + \frac{20Log(e)}{(r_o^2 + y^2)} y \qquad \text{(X.16)}$$

This provides a means of calibrating the test system. Generally the term $\frac{20Log(e)}{(r_o^2 + y^2)} y$ is very small for small y movements and the test field dB change is the dB change in S21 ie. $\Delta S21$. For larger y movement but with $r_o^2 + y^2 \sim r_o^2$ the test probe movement yields:

$$\frac{\partial G_t^{dB}(y)}{\partial y} = \frac{\partial S21(y)_{dB}}{\partial y} + \frac{20Log(e)}{r_o^2} y \qquad \text{(X.17)}$$

The dB change in path loss is linear with probe movement.
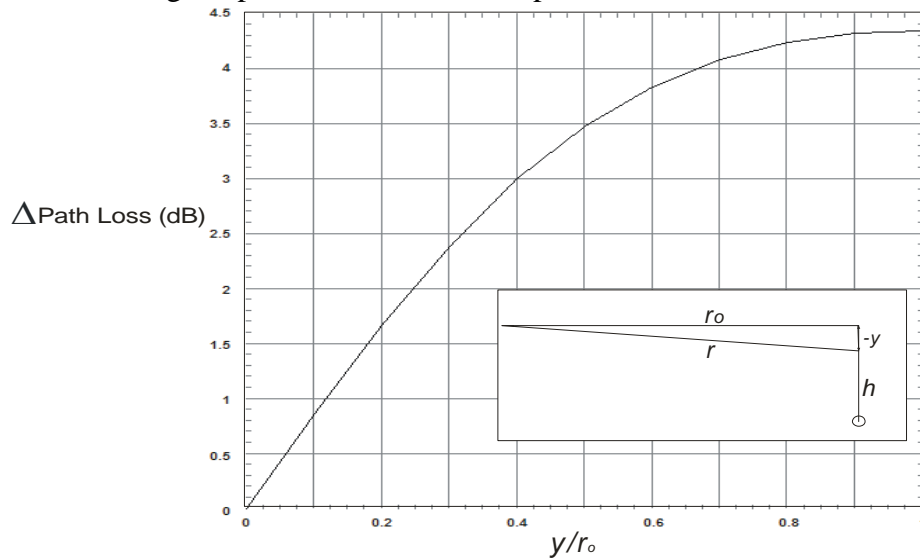
Fig 1-6 Path loss deviation for vertical movement of the test probe normalized to *ro=1*.

The field calibration consists of first aligning the phase centers of the source antenna with that of the isotropic probe. The probe is then lowered by a small amount and the dB change in S21 noted. Ideally the probe will be moved through the entire elevation range generating a data array which can be used in a measurement sequence as a correction matrix $[\Delta G_t]$.

Boresite

In antenna measurement it is necessary to have some point of reference. In the case of a simple measurement the data is taken as a function of Az and EL it is necessary to establish a reference angle for the AUT. The Az=EL=0 point is generally taken to be perpendicular to the AUT plane of symmetry in the quadrant of peak signal. The physical aspects of the AUT generally dictate the Az=EL=0 plane. However in practice peak signals and intuitive reference planes may not exactly be where they are supposed to be. High gain phased arrays require extremely accurate boresite alignment. Consider the Azimuth measurement shown below.

Simple antenna measurement over Azimuth with unknown distance r and boresite angle Φ
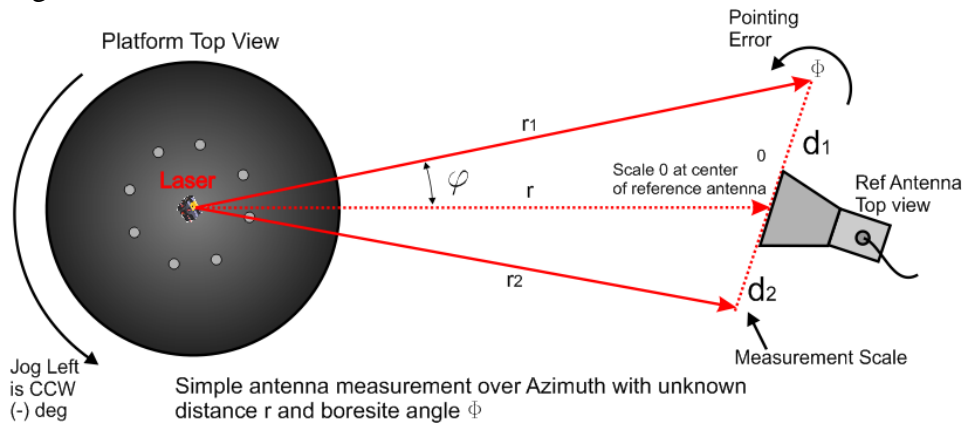
In this measurement a scale has been placed parallel to the AUT reference plane as defined by the manufacture. The scale origin is made to be precisely at the center AzEL point of the aperture. The AUT is replaced with a laser so that the scale is illuminated as the platform is rotated. Assume the platform is jogged + and - φ. The distances d1 and d2 are noted. By applying the Law of Cosines to both triangles and the entire periphery we have:

$$0 = -d_1^{\,2} + r_1^{\,2} + r^2 - 2r_1 r\cos(\varphi)$$
$$0 = -d_2^{\,2} + r_2^{\,2} + r^2 - 2r_2 r\cos(\varphi)$$
$$0 = -(d_1^{\,2} + d_2^{\,2}) + r_1^{\,2} + r_2^{\,2} - 2r_1 r_2 \cos(2\varphi)$$

The three equations can solved numerically for the unknowns r_1, r_2, and r. The tilt (boresite) angle can then be determined as:

$$\cos(\phi) = \frac{r^2 + d_i^2 - r_i^2}{2rd_i}$$

Where i denotes the upper or lower triangle. It is notable that the equations are not linear. If the first or second equation is solved for r_i we have

$$r_i = r\cos(\varphi) \pm \sqrt{r^2(\cos^2(\varphi) - 1) + d^2_i}$$

Since $r\cos(\varphi)$ is the projection on to the real axis the positive root must be selected in each case. However it is possible for the root to have a zero or negative argument leading to a maximum value of r given by

$$r_{max} = \frac{d_i}{\sqrt{(1 - \cos^2(\varphi)}}$$

When φ is small r_{max} can be used as an upper limit in numerical solutions. The DAMs laser algorithm can be used for measurement solutions. The case of perfect boresite ($d_1 = d_2$) forming a right triangle can be used to estimate distances and as a check to the numerical solution.

Example: Assume the distance and boresite accuracy is unknown. What is known is the platform jog angle and the distances d_1 and d_2.

$$\varphi = \pm 5 \deg$$
$$d_1 = 10.7$$
$$d_1 = 10.3$$

When the data is input to the DAMs calculator the following results:

The tilt angle 10.73 deg and the associated distance is 118.2". The right triangle solution shows that for r=118.2 d=10.34. We will assume the actual distance = 118.2. The Reference is then rotated and the data retaken. The measured distances were resolved to the nearest .1. The resulting distance 117.7 is within .2% of the actual distance. In an antenna gain measurement that would translate to 1.8m%.

Laser Alignment Tool

Link Distance And Pointing Algorithm

Purpose: Calculates the distance "r" and the pointing angle error for a reference antenna.

This routine will accurately calculate distance from the platform axis of rotation (rotary joint) to a surface remotely located. Below, d1 and d2 form a scale placed at the reference surface of the referance antenna. The scale 0 is at the geometric center of the horn(in this case). With the platform at Az=0 such that the laser is at the scale center 0, the platform is jogged a small angle Cw and CCW and the distances d1 and d2 are noted. Knowing the jog angle, d1,d2

Platform Top View

Calculate   Exit

Rotate CCW   -9.929m

Pointing Error

r_1 118.2

φ 5

Scale 0 at center of reference antenna

Distance = 117.7

r_2 118.2

Laser

Jog Left is CCW (-) deg

Φ

d_1 10.3

0

Ref Antenna Top view

d_2 10.3

Measurement Scale

Platform Range Estimator assuming perfect alignment

Calculate

φ 5

r

d   10.34

118.2

r

Phi

d

Measured calibration r=   118   Apply Cal point

Perfect alignment parameter calculator

Use the calculator to estimate d, r **or** Phi for perfect boresight alignment between laser and reference antenna. For example assume the ref antenna has been conveniently placed at say ~20 feet and boresight by visual estimation. The platform is rotated 5 deg and d is measured as 3 feet. Enter 3 in the "d" window and find the distance by pressing the r button as 34.39 ft. CALIBRATE PLATFORM s

1.5 Gain Transfer Method of Measurement

The gain transfer method of antenna measurement is the most widely used technique. Based on simple comparison it requires a calibrated reference antenna and knowledge of the path loss between the two antennas. The measurement is based on far field assumptions. The three primary field regions, the near field reactive region, the radiating near field and the far field. It can easily be shown that the inverse square law becomes a more complicated relationship at regions closer to the radiating elements.

Measurement point

R1

R

R2

Origin
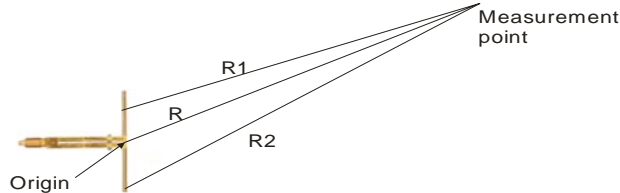
Fig 1-7 Radiating near field region where field strength S is $\vec{S}(R^{-2}, Rn)$, When R becomes large compared to the largest radiating element Rn ~ R becomes the far field region.

The transition from near field and far field is not a discrete distance and depends on the nature of the radiating structure. However equation 4.9 is a widely accepted relationship in terms of the largest dimension (*D)* of the radiating element of a large radiator.

$$R_{ff} > \frac{2D^2}{\lambda} \tag{1.18}$$

For antennas which are not electrically large such as dipoles whose lengths are on the order of $\lambda/2$ the transition region can be very close ie. $\frac{\lambda}{2}$ and the second condition (Eq. 4.10) is applied.

At regions very close to the radiating element non-propagating fields become dominant. Coupling to test probes may exist magnetically, electrically or both. These fields have varied applications such as RFID and RF components.

Now consider what happens when the path loss is evaluated for path lengths less than 1 wavelength. The path loss:

$$P_L = (\frac{\lambda}{4\pi r})^2$$

Is indeterminate when r = 0. One expects the path loss to be a number between 0 and 1. We solve for the condition where

$$P_L = (\frac{\lambda}{4\pi r})^2 > 1$$

for which $r < \frac{\lambda}{4\pi}$ about 1/12th wavelength

If r is less than this then the path loss becomes a gain, an impossible situation. The solution to the problem stems from the fact that as the path loss is reduced to inside the near field where free photons are generated, the link begins to miss free photons thereby reducing the gain. The measured link gain must approach
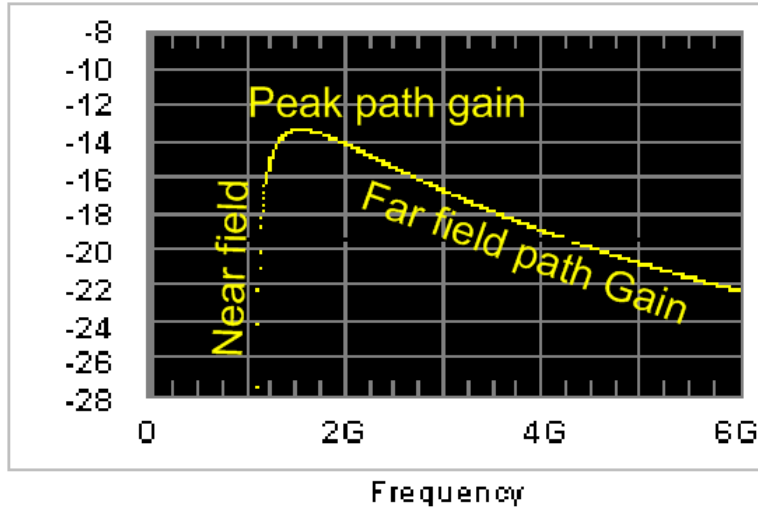
$$LinkGain \to 1 = (\frac{\lambda}{4\pi r})^2 G_{Tx} G_{Rx} |_{r \to 0}$$

This results in the product $G_{Tx}G_{Rx}$ reducing so as to directly compensate the . Corrections can be made for specific cases by utilizing the near field variations. In the case of electric antennas the path loss is defined as

$$(\frac{\lambda}{4\pi r})^2 - (\frac{\lambda}{4\pi r})^4 + (\frac{\lambda}{4\pi r})^6 \quad 1.186$$
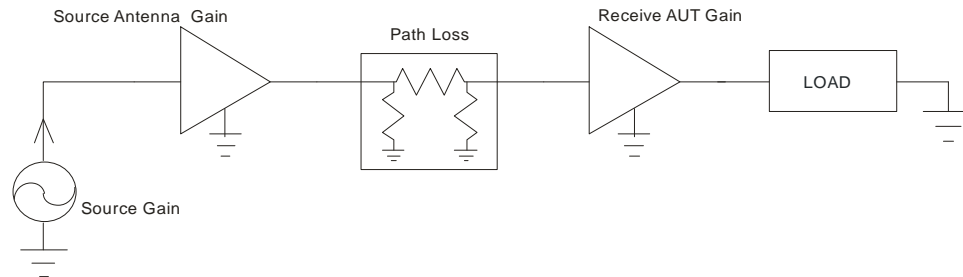
If eq. 1.186 is evaluated for small to large $\frac{r}{\lambda}$ from 1 to 6GHz we get

Path loss for very small $\frac{r}{\lambda}$

The above equations in conjunction with dipole geometry form the plot.

Referring to Eq. 1.2 it is instructive to establish the entire measurement link as a 3 element system.

Fig 1-8 AUT (Antenna under test) measurement test setup for Gain Transfer Technique.

Figure 1-8 represents a simple 3 element network where two of the three elements are known and the third (AUT) is to be determined. Solving Eq. 1.2 for G_r^{dB} yields

$$G_r^{dB} = S21_{dB} - P_L^{dB} - G_t^{dB} \tag{1.19}$$

The path loss is determined from Eq. 1.6 and the Source Antenna Gain is supplied from the manufacture.

$$P_L^{dB}(d = r) = 20 Log(\frac{\lambda}{4\pi r}) \tag{1.6}$$

Measurement Example – Gain Transfer

In this example a 5.8GHz double helix antenna is to be measured over a
frequency band extending from 5GHz to 5.9GHz. Fig. 1-8 shows a typical
platform consisting of two movement stepper motors an acrylic test
platform and a rotary joint. The platform movement is software controlled
and interfaces to a Vector Network Analyzer(Fig. 1-9).
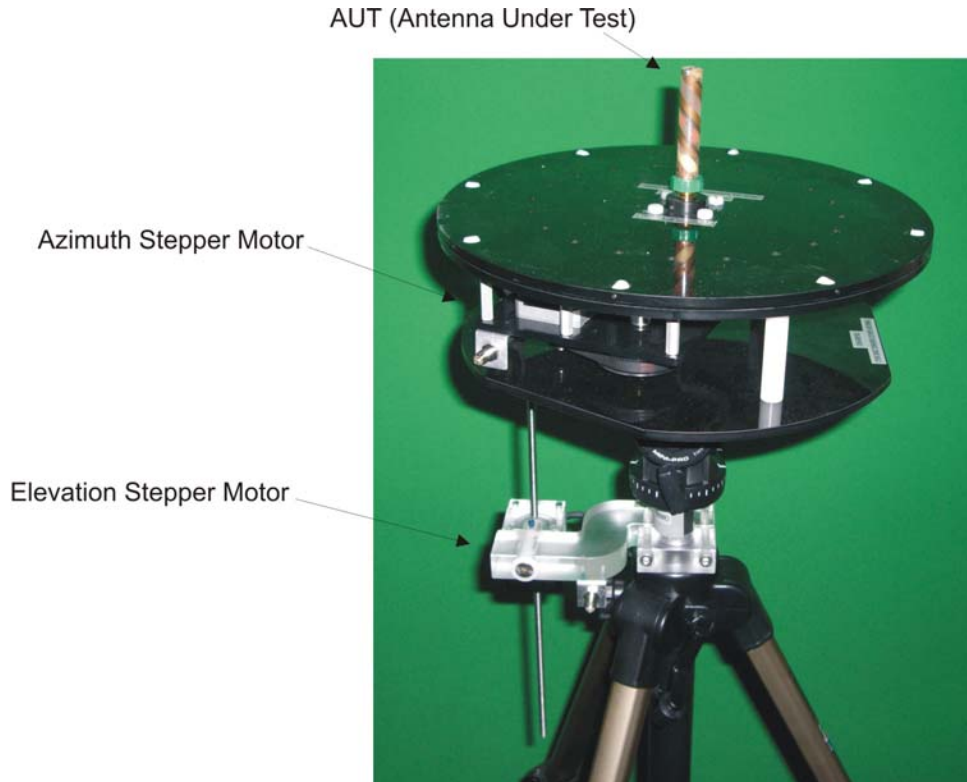
Fig 1-8 Commercial antenna platform used for Azimuth and Elevation rotation driven by software and
associated measurement equipment. Courtesy of Diamond Engineering Inc.

The most common control interface is RS232 serial. Compatibility with
modern USB interface is achieved through USB to Serial converters. The
platform Azimuth movement is achieved through geared stepped rotation
of the upper plate. The elevation is changed using the Elevation threaded
pushrod. The above platform is easily capable of ± 45 degrees Elevation in
.1deg increments and 0-360 deg Azimuth in .1 deg increments. Fig1-8.1
shows a typical antenna test system interface.
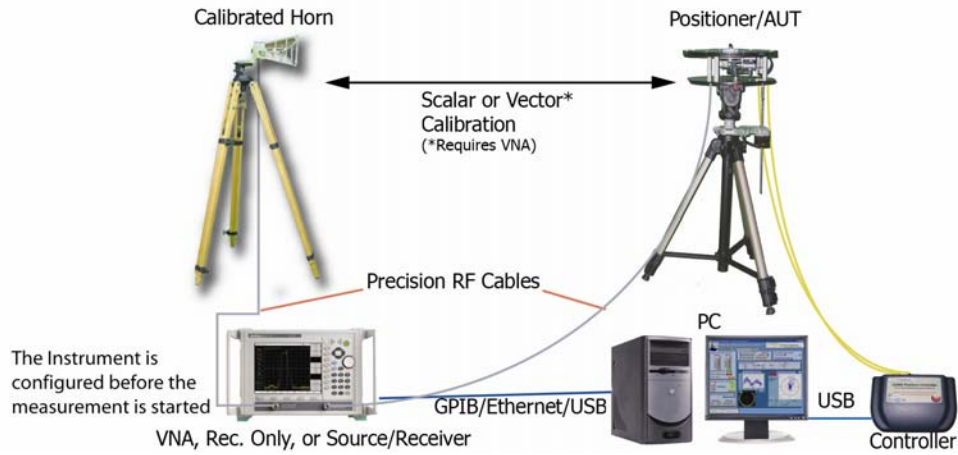
Fig 1-8.1 A typical measurement setup using a vector Network Analyzer, stationary calibrated reference horn, controller PC and a programmable platform. The PC instructs the positioner to move, triggers a frequency sweep from the VNA and adds the data to a software measurement array.

Simple test platforms can be driven from special microwave software to produce some amazing plots such as the spherical plots below.
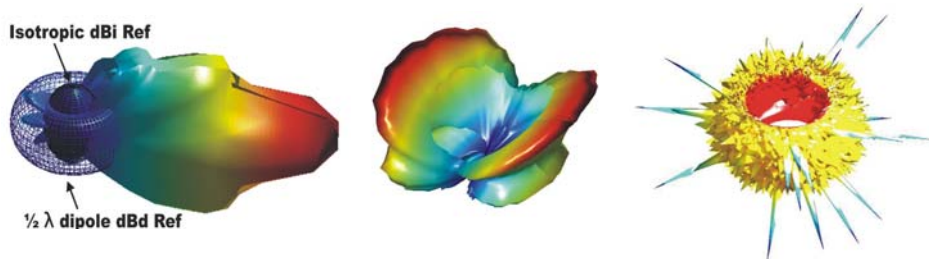
Fig 1-8.2 Actual Az-El measurement data with Cartesian to Spherical transformations applied.
    A.) Patch antenna with ideal isotropic and dipole references.
    B.) Long-wave dipole.
    C.) ¼ wave dipole log response showing wireless network interference.

As previously demonstrated the elevation "swing" of the antenna can affect measurement accuracy (Fig 1.2). Another more prevalent problem is that of multi-path interference. The possibility that reflected rays from other objects may be incident on the AUT. The effect is shown in Fig1-9.
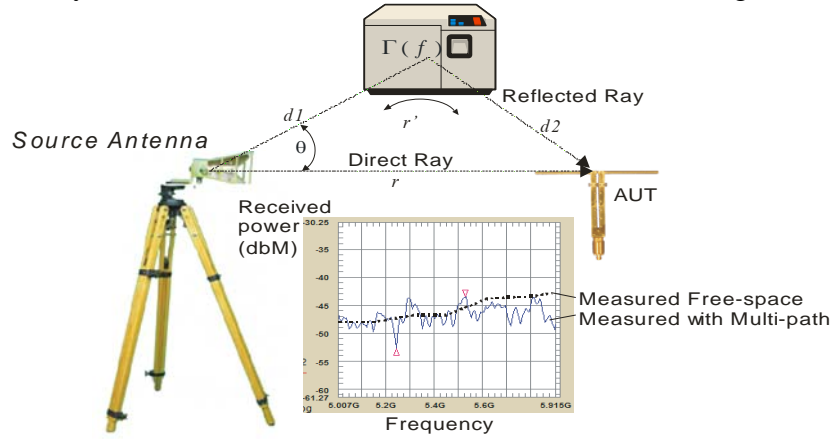
Fig 1-9 Multi-path effect in antenna measurement systems caused by reflections from nearby objects. The reflected ray travels a longer path and is attenuated by the reflection cross-section. Result is that ripples occur in the received power profile.

As can be seen by the power variation (fig. 1-9), multi-path can inject an immense variation in the received power rendering measurements meaningless. For this reason anechoic chambers consisting of carbon based absorber covering the walls are used.

Fig 1-10 Anechoic antenna measurement chamber constructed for very high microwave frequency antenna measurements (Courtesy of Motorola)

<u>Making measurements without the use of an anechoic chamber.</u>
Examination of fig. 1.9 and equation 1.5 can suggest a simple method for measuring antennas in a cluttered environment. One first notes that if $\frac{\lambda}{d} << 1$ (as it is in the far field) the path loss gradient becomes small. This means that even though the multipath ray may travel considerably further distance, the path loss may be approximately the same. This explains the extreme ripples seen in the received power.

$$P_L^{dB} = -21.98 + 20Log(\lambda) - 20Log(d) \tag{1.5}$$

One method is to simply reduce the measurement distance until the variation is less than the desired accuracy.
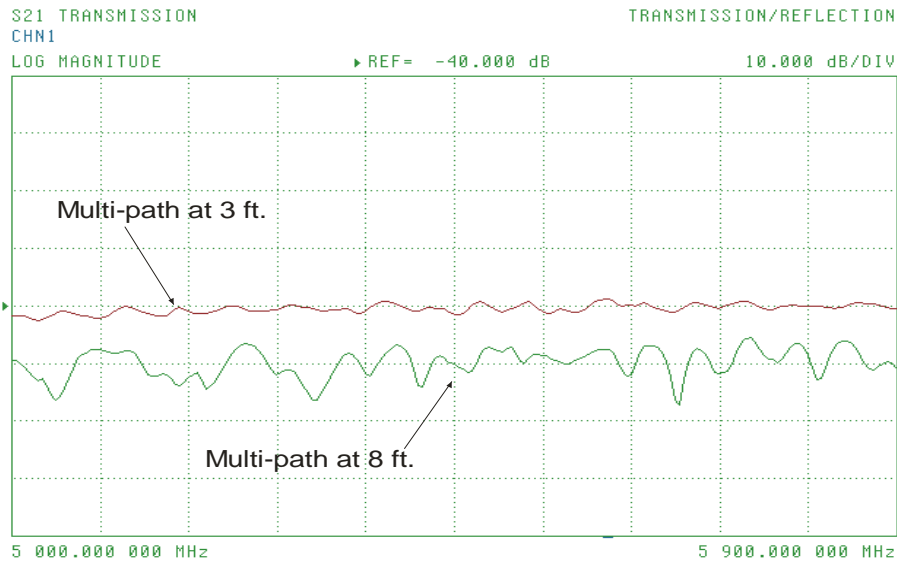

Fig 1-11 Reducing the affect of multi-path by decreasing the measurement distance

The gain of a circularly polarized helix antenna(Fig 1-8) is to be measured in the vertical direction over a frequency range of 5GHz to 5.9 GHz. The helix is intended for use over the 5.8GHz band and has a rated gain of 3.1dB. A broad band calibrated horn is used to supply the test signal from a vector network analyzer(vna). The measurement summary is as follows:

- AUT   Helix circularly polarized
- Freq   5.0 to 5.9GHz
- Separation   36 inches

- Azimuth   0-360 deg 10deg steps Vertical orientation
- Source   Anritsu MS4623B Vector Analyzer
- Tx   Calibrated Horn

In any measurement system it is necessary to compensate for the loss of the test system. In this case the system consists of RF cables and the AUT platform. Two common methods of system calibration are the Scalar cal and the Vector cal. The scalar cal requires knowledge of only the system loss while the vector cal requires calibrated standards and complex correction math. The vector cal provides phase as well as amplitude measurement. Phase measurement can be used to calculate the group delay to determine the system separation distance. In addition vector measurements allow the use of special transforms for data conversion to time domain or minimum phase.
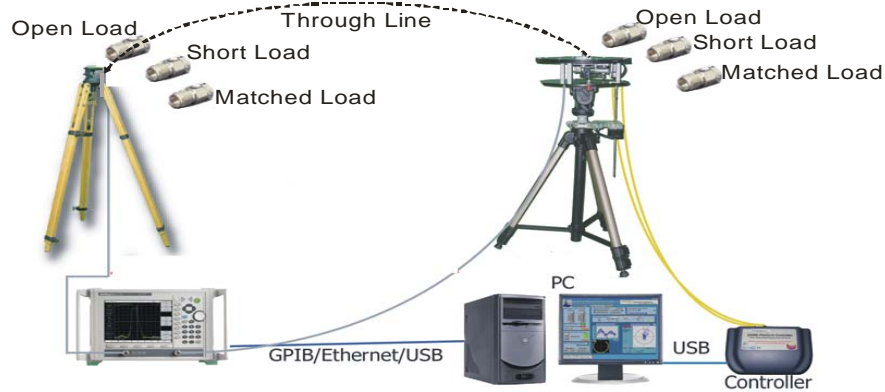
Fig 1-13 Vector calibration utilizing Open-Short-Load and through lines to remove the frequency response of the measurement system

The measurement steps are as follows:
- Calibrate the system loss
- Measure: Reference Antenna(dB) + Path Loss(dB) + AUT(dB)
- Calculate the path loss (Eq. 1.5)
- Calculate the AUT gain(dB) $G_r^{dB} = S21_{dB} - P_L^{dB} - G_t^{dB}$
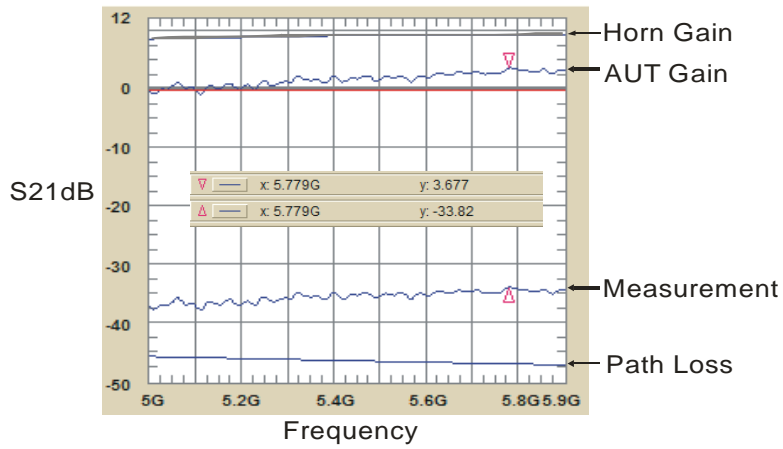
**Fig 1-14** Antenna gain measurement over frequency showing measurement levels

The AUT gain is approximately 3.7dB. Since the AUT was rotated through one complete azimuth revolution the gain may experience change. Viewing the gain calculation as a function of rotation at a fixed frequency in polar coordinates is generally preferred.
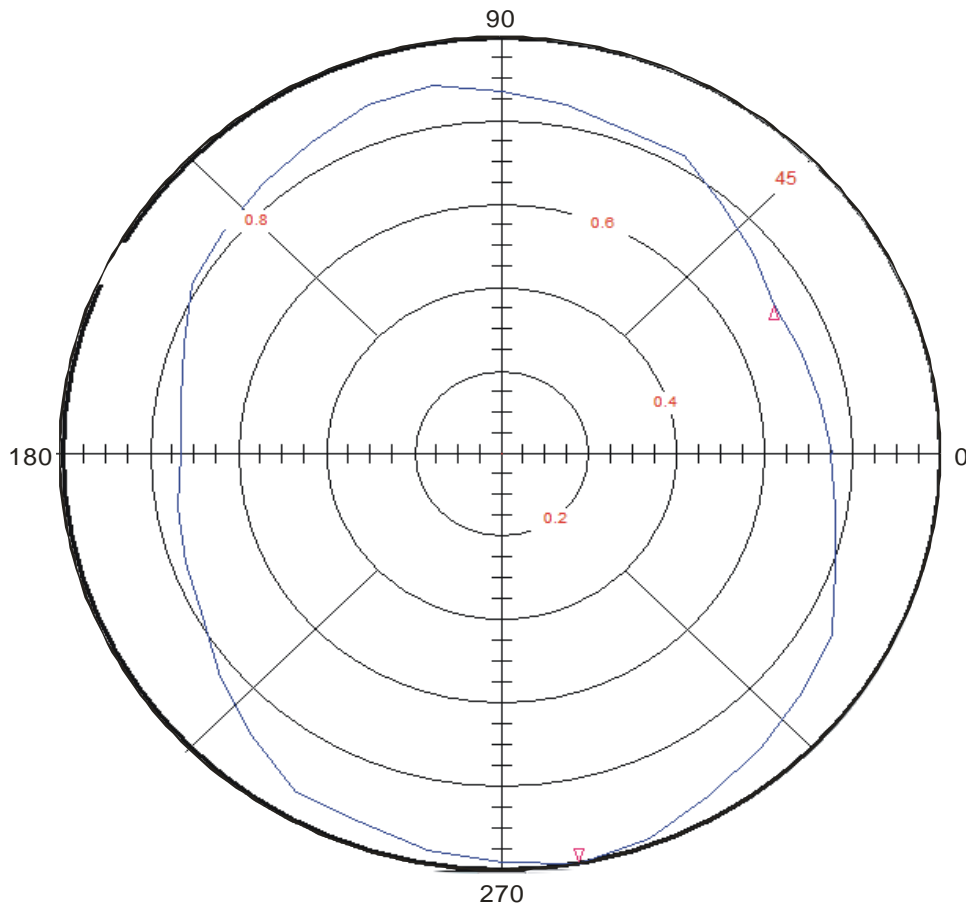
Fig 1-15 Polar representation of antenna gain measurement normalized to Gmax.

Since most receive systems operate over a wide dynamic range a log plot is useful. The linear polar plot of fig 1-15 may also be represented in Log format. Generally a linear axis is employed. When used, a singularity is present as $G \rightarrow 0$. For this reason the log plot does not extend inside the first circle.
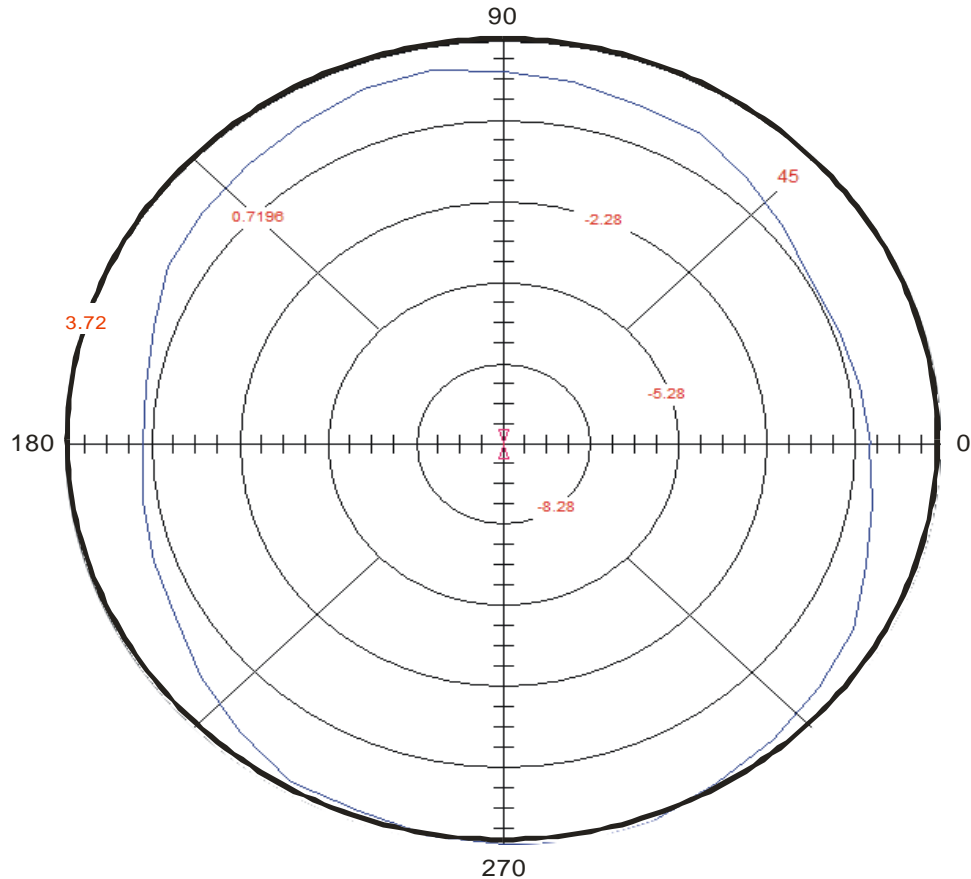
Fig 1-16 Log polar representation of antenna gain at 3dB/div. Outermost circle represents Gmax = 3.72dB. Span is 12dB (to inner circle).

The log plot shows the dB variation of the AUT as it rotates. This can be useful to determine omni characteristics or beam width of high gain antennas.

1.6 3-Point Method of Measurement

The 3-point technique establishes Friis link equation as three equations with three unknowns. The arrangement is shown in Fig 1-17. The measurement pairs are [AUTa,AUTb], [AUTa,AUTc] ,[AUTb,AUTc] giving the following set of equations:

$$[S21_{ab}] = [P_L] + [G_a] + [G_b]$$

$$[S21_{ac}] = [P_L] + [G_a] + [G_c]$$

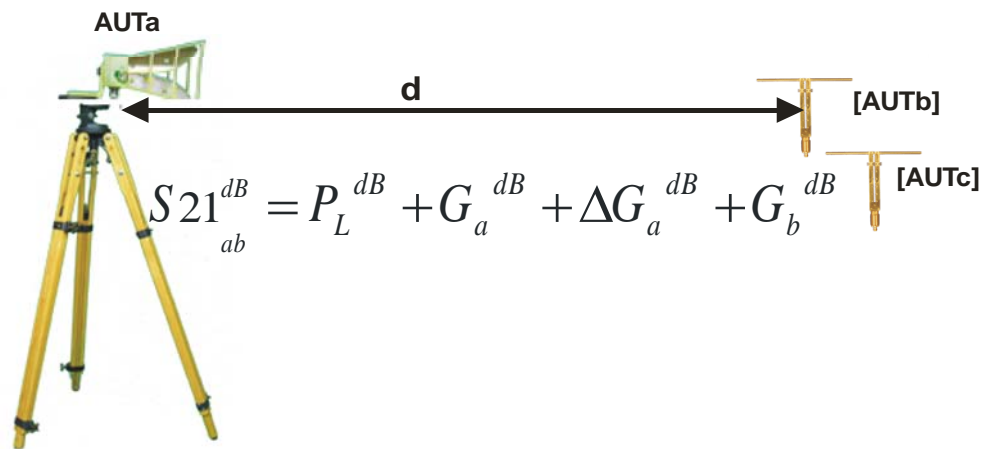$$[S21_{bc}] = [P_L] + [G_b] + [G_c]$$

The brackets denote Az-El measurement data sets. The solution for the gains is:

$$[G_a] = \frac{[S21_{ab}] + [S21_{ac}] - [S21_{bc}] - [P_L]}{2}$$

$$[G_b] = [G_a] + [S21_{bc}] - [S21_{ac}]$$

$$[G_c] = [S21_{bc}] - [P_L] - [G_b]$$

where all array values are in dB.

AUTa

d

[AUTb]

[AUTc]

$$S21_{ab}^{dB} = P_L^{dB} + G_a^{dB} + \Delta G_a^{dB} + G_b^{dB}$$

Fig 1-17 Three point method utilizing all combinations of three antennas to establish 3 equations with 3 unknowns

While the solution seems straightforward there can be a subtle problem when only two AUTs are measured over Az-El extents. Generally AUTa is stationary. AUTb is then measured over Az-El extents. AUTc is measured over the same Az-El extents. Then AUTa is replaced with AUTb. The solution at each point of movement requires both AUTb and AUTc simultaneously move. Generally this is not possible with a single movable platform. If the AUT movement is restricted to Az only and AUTb is symmetrical as with a dipole then substitution of AUTb for AUTa does not require rotation of AUTb. In this case the tolerance of AUTb symmetry will sum into the measurement accuracy.
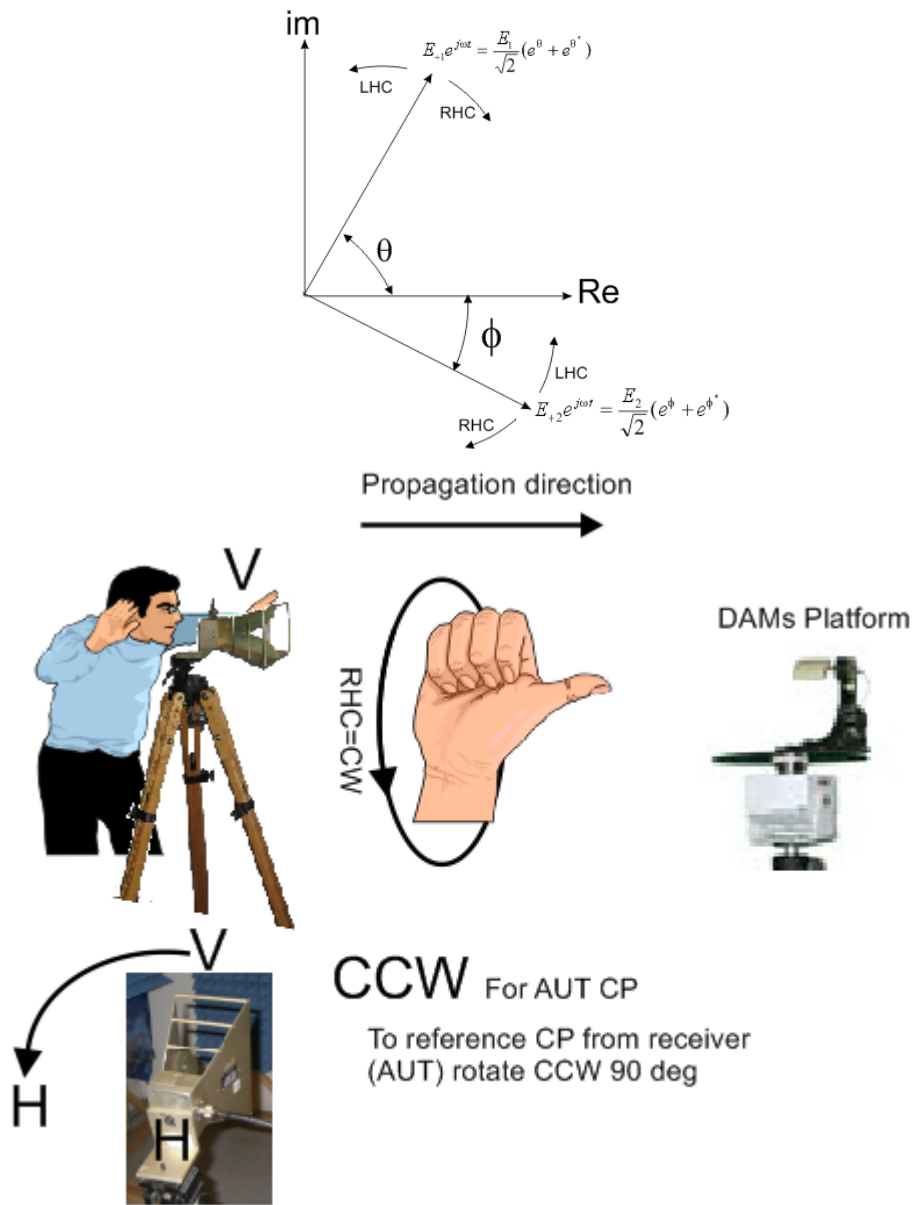
1.7 Circularly Polarized Measurement

The most popular and simplest method of measuring a CP antenna is the HV method. Assuming the AUT is circular or elliptically polarized the reference antenna is used to make two measurements, one positioned vertically and one measurement positioned horizontally. The results are then combined to determine the LHP and RHP gains. To see how this works requires knowledge of complex numbers. It is well known that any linear vector can be the sum of two counter rotating vectors.

$$E_{+1}e^{j\omega t} = \frac{E_1}{\sqrt{2}}(e^{\theta} + e^{\theta^*})$$
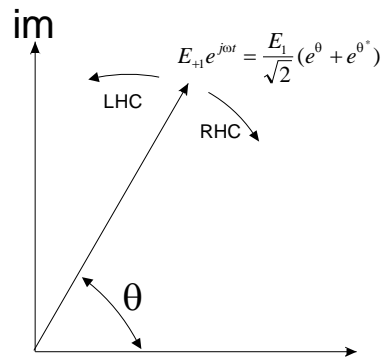
Fig 1-18 A single linear vector is equivalent to two counter rotating vectors each with reduced amplitude

With circular the question of reference generates multiple standards. Generally the right hand rule can be applied. However in a test system the aut of interest may be receiving not transmitting. That is the case with most aut test systems. For the DAMs system the direction of propagation is toward the aut and the observer is assumed stationed at the Tx. RHC is then clockwise. To reference CP to the AUT the rotation direction is changed to CCW. This is easily seen from the Real Imaginary propagation vector. If the reference horn is rotated CCW with respect to the RHC direction the result is opposite rotation of the propagation vector.

$$E_{-1}e^{j\omega t} = \frac{E_1}{\sqrt{2}}(e^\theta + e^{\theta^*})$$

LHC

RHC

im

θ

Re

φ

LHC

$$E_{+2}e^{j\omega t} = \frac{E_2}{\sqrt{2}}(e^\phi + e^{\phi^*})$$

RHC

Propagation direction

RHC=CW

V

DAMs Platform

V

H

CCW For AUT CP

To reference CP from receiver
(AUT) rotate CCW 90 deg

Fig 1-19 The right hand rule. With the thumb in the direction of propagation RHC follows the fingers. In this case the observer sees RHC as clockwise rotation from the source but CCW must be used for the AUT

Let us assume the H measurement result is $E_1 = E_{+1}e^{j\theta}$ and the V measurement is $E_2 = E_{+2}e^{j\phi}$. It is necessary have phase information such as that provided by a vna. The result of the vector addition of these two measurements in terms of two counter rotating vectors is:

$$LHC = \frac{1}{\sqrt{2}}(E_{+1}e^{j\theta} - jE_{+2}e^{j\phi})$$

$$RHC = \frac{1}{\sqrt{2}}(E_{+1}e^{j\theta} + jE_{+2}e^{j\phi})$$

Then each vector is broken down into real and imaginary as:

$$RE(LHC) = \frac{1}{\sqrt{2}}(E_{+1}\cos(\theta) + E_{+2}\sin(\phi))$$

$$IM(LHC) = \frac{1}{\sqrt{2}}(E_{+1}\sin(\theta) - E_{+2}\cos(\phi))$$

$$RE(RHC) = \frac{1}{\sqrt{2}}(E_{+1}\cos(\theta) - E_{+2}\sin(\phi))$$

$$IM(RHC) = \frac{1}{\sqrt{2}}(E_{+1}\sin(\theta) + E_{+2}\cos(\phi))$$

The associated Phase angles (90 degrees ideally) are:

$$\theta' = \tan^{-1}(\frac{E_{+1}\sin(\theta) - E_{+2}\cos(\phi)}{E_{+1}\cos(\theta) + E_{+2}\sin(\phi)})$$

$$\theta' = \tan^{-1}(\frac{E_{+1}\sin(\theta) + E_{+2}\cos(\phi)}{E_{+1}\cos(\theta) - E_{+2}\sin(\phi)})$$

It should be noted that positive reference antenna rotation should follow the right hand rule to prevent inadvertently switching RHC and LHC.

1.8 Antenna Efficiency
Antenna efficiency is generally defined as the ratio of total received power to transmitted power supplied to the Tx antenna. Measurement of total received power must be done by scanning the entire sphere enclosing the source or vice versa. The received power at each point is:

$$P_r = P_{TX} P_L G_T G_r (\theta, \phi) \qquad (1.6.1)$$

Where the Tx antenna position is constant and the Rx aut is rotated about it's phase center. The aut is presumed to be dimensionally small.

The link efficiency, $\varepsilon = \dfrac{P_r}{P_{TX}}$, can be calculated as:

$$\varepsilon = \frac{\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2\pi} P_d G_r (\theta, \phi) R^2 Cos(\phi) d\theta d\phi}{P_{TX}} \qquad (1.6.2)$$

Where Pd = Transmitted power density at a distance R
Pr = H+V Received power

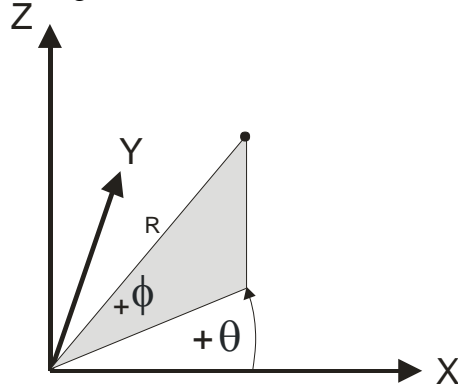$$Pd = \frac{P_{TX}}{4\pi R^2} \qquad (1.6.3)$$

The receive gain is determined from the measurement S_{21}^2 as

$$G_r = \frac{S_{21}^2 (\theta, \phi)}{P_L G_T} \qquad (1.6.4)$$

Substitution of 1.6.3 and 1.6.4 into 1.6.2 gives

$$\varepsilon = \frac{\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{2\pi} S_{21}^{2}(\theta,\phi)Cos(\phi)d\theta d\phi}{4\pi P_{L}G_{T}} \tag{1.6.5}$$

The choice of spherical coordinate system as shown below requires ϕ to range from -90 to +90 degrees.

If the Azimuth is swept from 0 to 360 degrees in N steps and the elevation swept from $-\frac{\pi}{2} to \frac{\pi}{2}$ in M steps then $d\theta$ can be approximated by $\frac{2\pi}{N}$ and $d\phi$ by $\frac{\pi}{M}$. If N and M are sufficiently large so that Pr per unit of area is constant the integration can be summed as:

$$\varepsilon = \frac{2\pi^{2}}{NM}\sum_{N}\sum_{M}\frac{S_{21}^{2}(\theta_{M},\phi_{N})}{4\pi P_{L}G_{T}}Cos(\phi_{N})$$

$$\varepsilon = \frac{\pi}{2NM} \sum_{N} \sum_{M} \frac{S_{21}^2(\theta_M,\phi_N)}{P_L G_T} Cos(\phi_N) \qquad (1.6.6)$$

Equation1.66 is the classical method of efficiency measurement in terms of S21 and referenced to level elevation.

Assume an isotropic link is supplied with 1 Watt of power and at a distance R the Rx efficiency is to be measured by Azimuth 5 degree movement and elevation 5 degree movement. In this case
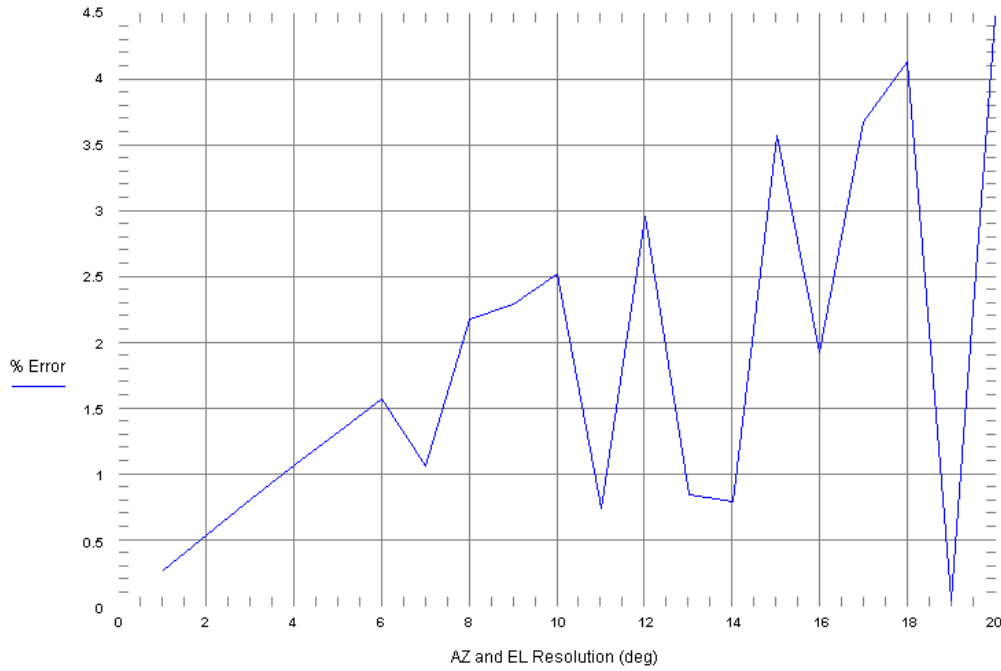
$$S_{21}^2(\theta_M,\phi_N) = \frac{1}{P_L}$$

And
$$G_T = 1$$

$$\varepsilon = \frac{\pi}{2NM} \sum_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sum_{0}^{2\pi} Cos(\phi_N)$$

Performing the sum yields:

$$\varepsilon = 101.3\%$$

A 1.3% error associated with 5 degree AzEL resolution. The question of how much error is experienced vs AzEL measurement resolution can be calculated from the isotropic link. The results are:
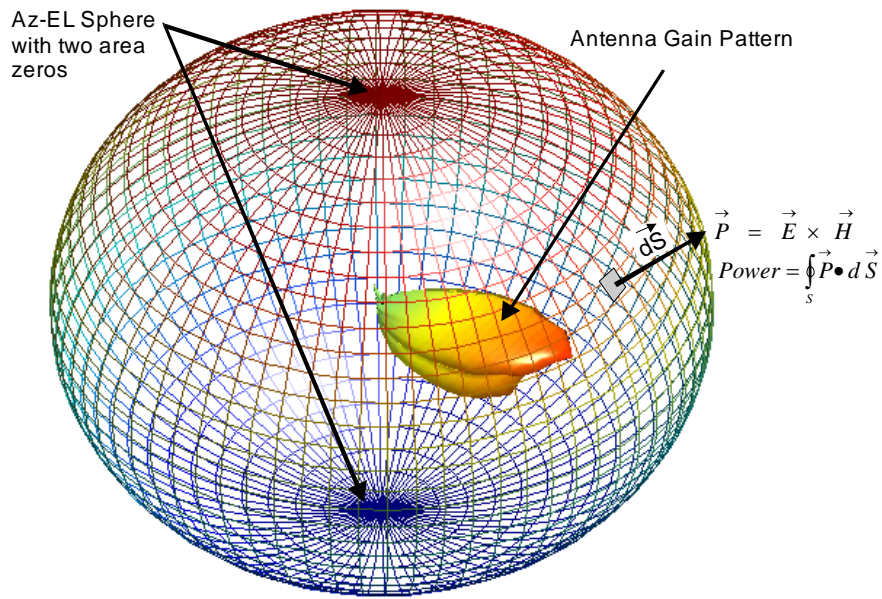
Fig 1-18 Efficiency measurement error vs Az EL resolution. For this case the Az steps = EL Steps from 1 to 20deg

As would be expected the measurement error increases linearly. However at 7,11,13….. degrees, the degree per cut is not an integer in both the Az and EL range. This error reduces the resolution error. It suggests that a spherical scan at 19 degrees resolution would be at least as accurate as at 1 degree resolution. A link with a patch antenna might have a beamwidth of 60 deg. A measurement resolution at 11 deg would reduce the error and make measurements faster.

Isotropic and Omni Efficiency Measurement

There are some positioning considerations when making efficiency measurements. When measuring an isotropic type radiator the power density at a distance r in spherical coordinates is;

$$E(r,\theta,\phi) = \frac{e^{jkr}}{4\pi r}\hat{i}(\theta,\phi)$$

Az-EL Sphere
with two area
zeros

Antenna Gain Pattern

$$\vec{P} = \vec{E} \times \vec{H}$$

$$Power = \oint_S \vec{P} \bullet d\vec{S}$$

$d\vec{S}$

Fig 1-19 Efficiency measurement based on Poynting vector integration over a spherical surface. Area ds diminishes to zero at top and bottom of sphere.

Single AzEL zero

AUT center
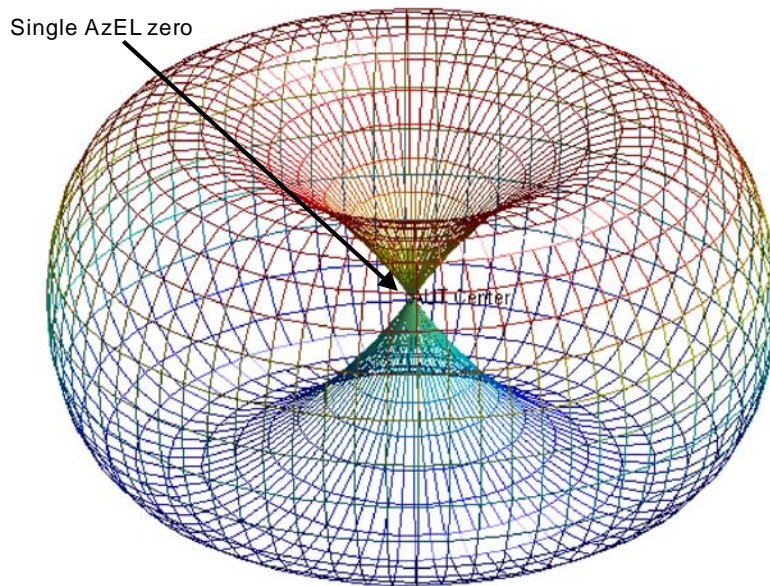
Fig 1-20 Dipole pattern with a single AzEL zero point

$$Power = \oint_S (\vec{E} \, X \, \vec{H}) \bullet d\vec{S}$$

1.9 Diamond Engineering Continuous Platform Rotation Measurements
Remote Location

The measurement of antenna pattern traditionally requires AUT start stop and step. The aut is motionless while the data acquisition is performed. Range is limited by the need for RF and acquisition cables. This problem can be overcome by applying Nyquist's classical sampling theorem and eliminate the need for triggering and cables. The question is "how fast can the platform rotate and still produce accurate data".

First assume the vna sweep speed is τ seconds. Assume the data S21(f) is transferred in a period T seconds and AUT is rotating at ω_P radians per second.
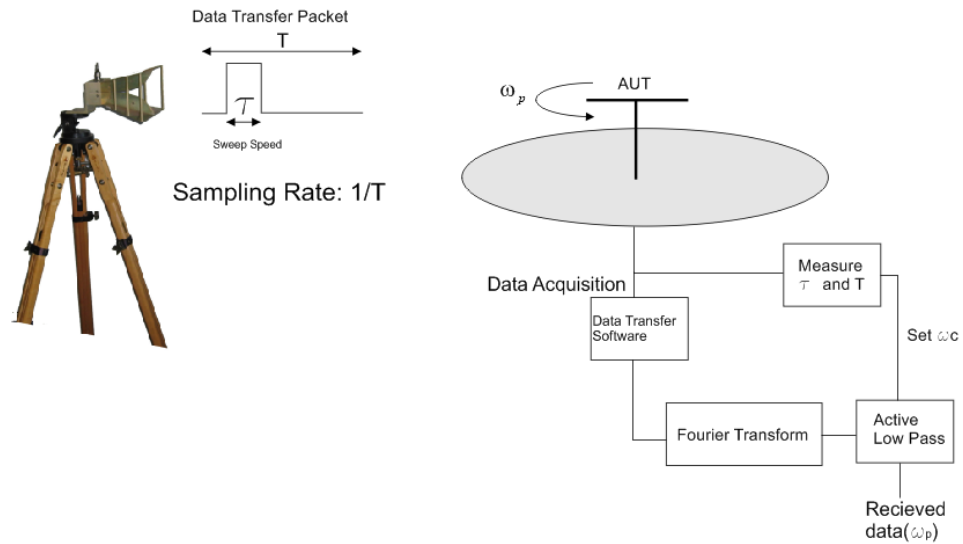
Fig 1-21 Remote measurement of antenna properties. Sampling period and width are initially determined to set the low pass filter cutoff frequency. The sample width is the sweep period while the sample rate is the data transfer frequency

The sample process is a direct application of the sampling theorem.
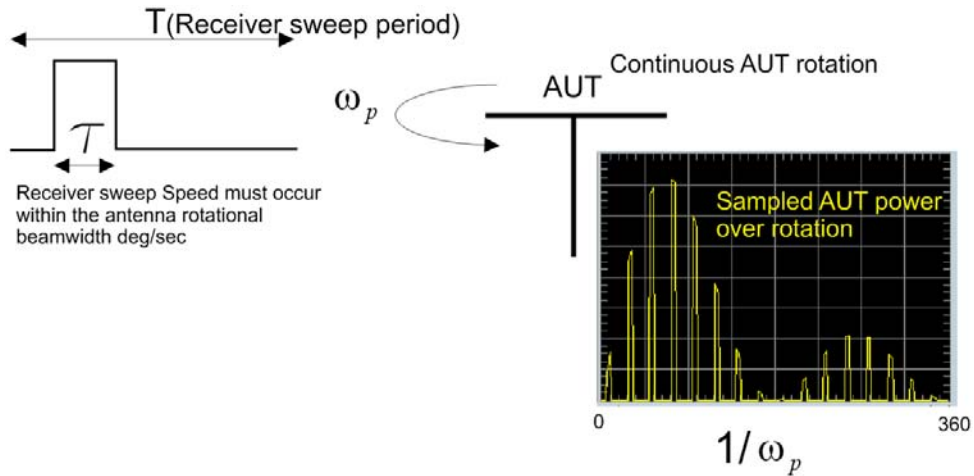
Fig 1-22 Sampled data for received power over Azimuth rotation.

The classical Nyquist sampling rate is:

$$Fs > 2B$$

Where B is the rotational bandwidth of the received signal. Generally a swept source and a swept receiver provide the necessary sampling. The sweep time is τ and the period T form the sampling source. The period T includes retrace time and data transfer time over a bus. The gain function $G(A_Z(t))$ of the AUT, subjected to periodic motion, form the signal to be sampled and recovered. Clearly the rate at which the gain function changes with angle forms the rotational bandwidth of the received signal. For normal rotation $A_Z(t) = \omega_P t$ where $\omega_P = \frac{\pi}{30} RPM$ radian cycles/sec.

The time dependence of the received signal power is

$$\Pr(t) = Pt(\tau)G(\omega_P t)$$

The gain function can be described as a shape over periodic rotation at a rate ω_P. Most gain shapes will have regular forms which mean equivalent rectangular beam widths are applicable.

BandWidth = 1/(2F$_P$DC) = 180/(F$_P$Beamwidth)

Azimuth Deg = F$_P$t
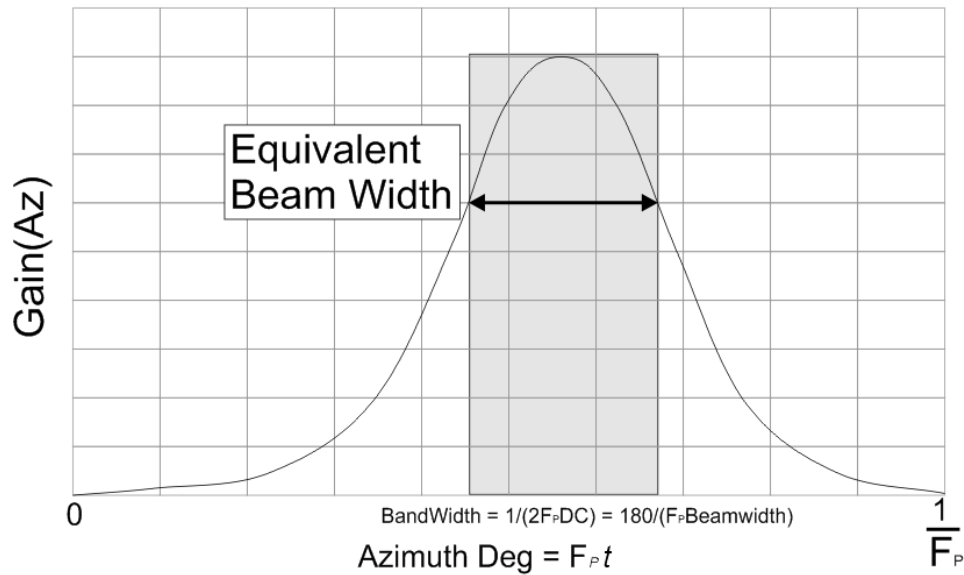
Fig 1-22 Time pulse equivalent of AUT rotation generating an approximate
beam width/duty cycle (DC).

The amplitude envelope for this problem is easily established as

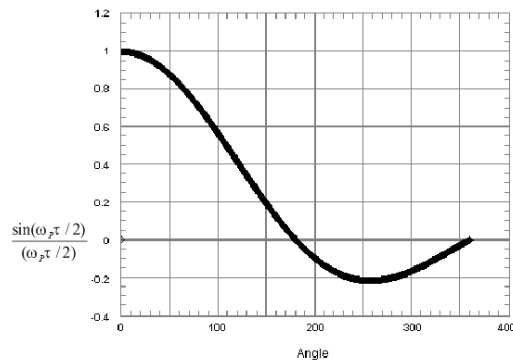$$\frac{\sin(\omega_p \tau / 2)}{(\omega_p \tau / 2)}$$

Where $\tau = EquivalantBeamwidth$

$\omega_P = PlatformSpeed$

A plot of the function shows the complex Fourier bandwidth and that

Fig 1-23   Amplitude-frequency envelope of the sampling pulse

Most of the energy is between 0 and (π) 180 degrees. It is reasonable to assume bandwidth can accurately be estimated by $\dfrac{1}{\tau}$ where τ is the equivalent energy pulse of the AUT gain shape.

If the platform is rotating at RPM then the rotation frequency is $\dfrac{RPM}{60}$ and the associated period is $T = \dfrac{60}{RPM}$. The period associated with the beamwidth is simply

$$\tau = \frac{60}{RPM} \times \frac{Beamwidth}{360} = \frac{Beamwith}{6RPM}$$

The bandwidth is then:

$$BW = \frac{k}{\tau} = \frac{6RPM}{Beamwidth} \quad \text{Where } k \text{ is 1 for } \tau << T$$

The Nyquist minimum sampling rate is

$$F_S = 2 * BW = \frac{12 * RPM}{Beamwidth}$$

The platform rotational frequency is $f_p = \dfrac{RPM}{60}$ cps and the minimum sampling rate is then
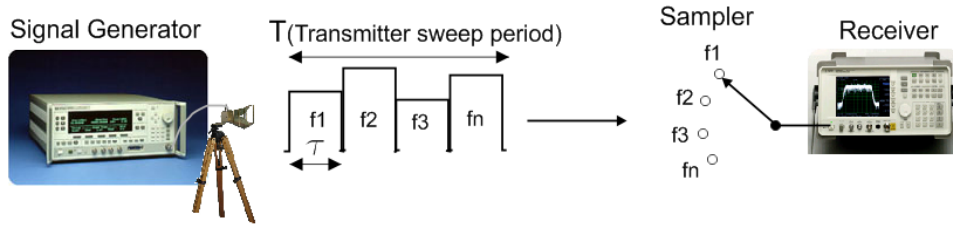
$$Fs = \frac{12 * RPM}{Beamwidth} \quad\quad \text{where} \quad Fs = cycles/\sec$$

$$Beamwidth = \deg$$

The receiver needs to sample at rate $\dfrac{1}{Fs}$ samples/sec.

Now consider a non-synchronous wireless system. The Tx signal generator continually outputs frequency pulses with negligible delay and no retrace delay.

Non-syncrounous frequency channel sampling system

On the receive side a multiplexer is used as a sampler. A spectrum analyzer is ideal for serving as the multiplexer and receiver. If the system were synchronous the analyzer sweep speed would need to be $n\tau$. Because the sampler is free running one needs to insure all frequencies are received by setting the sweep speed to τ. Otherwise it will be necessary to make multiple 360 deg rotations.

Given the platform rotational speed and the AUT beamwidth (or the desired resolution beamwidth) the transmit period will need to be:

$$T = \frac{1}{Fs} = \frac{Beamwidth}{12 * RPM}$$

And the receiver sweep speed

$$T = \frac{1}{nFs} = \frac{Beamwidth}{12n * RPM}$$

As an example assume 51 frequencies are transmitted to a receive antenna with a 45 deg beamwidth. It is desired to set the measurement resolution accuracy to 5deg. The rotating measurement platform rotates 360 deg in 1 minute.
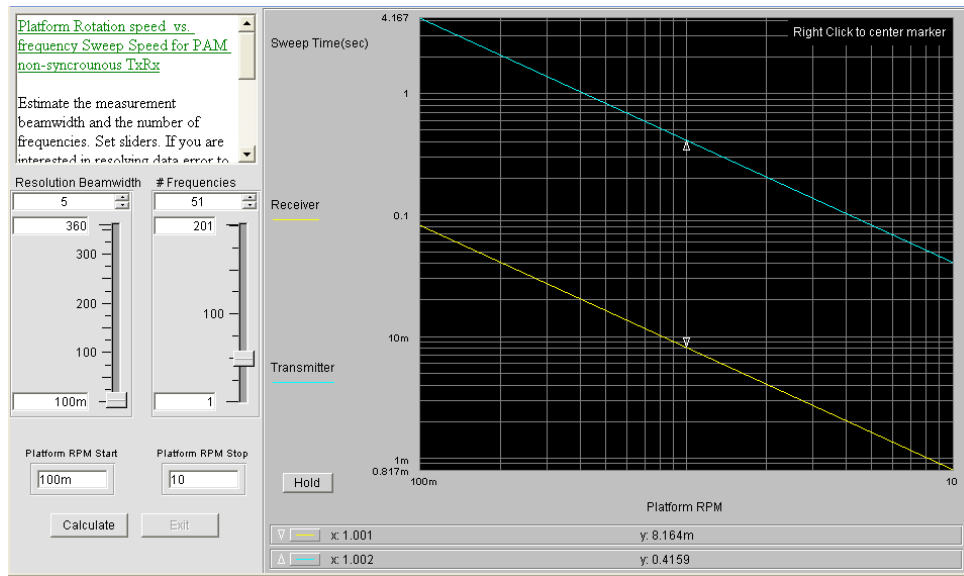
In this case the maximum transmit period will need to be set to:

$$T = \frac{Beamwidth}{12 * RPM} = \frac{5}{12} = 417 m\sec$$

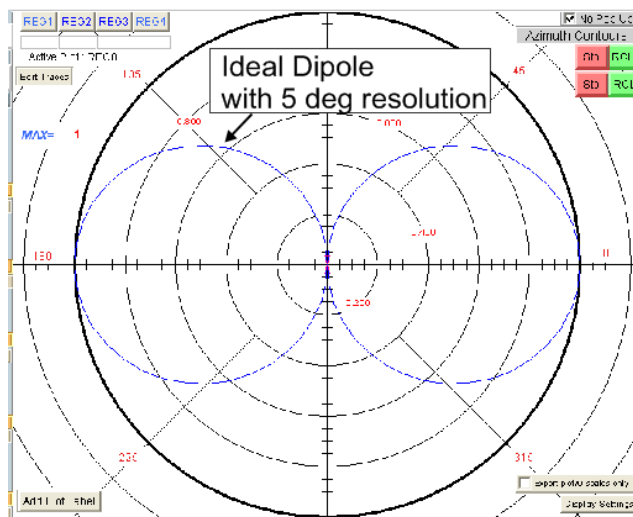And the receiver minimum sweep speed is then $417/51 = 8.17ms$

Using the DAMs sampling software a plot of Sweep(s) vs PlatformRPM yields:

Example: Error reduction

As an example of using the sampling system to eliminate error we first analyze an ideal dipole over an Az range of 0 to 360deg with 5 deg resolution.

The data is then given random amplitude error with +/- 25% variation. The embedded error file is then processed through the sampler to 1deg resolution and compared with the original response. From the practical dipole equations it is known that sampling to order 6 reproduces the dipole equations accurately.

Ideal dipole with
random error

Sampled output
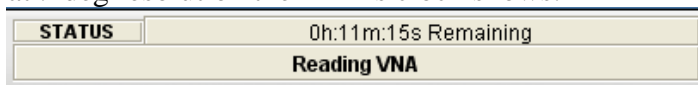at 1 deg resolution
with order 6
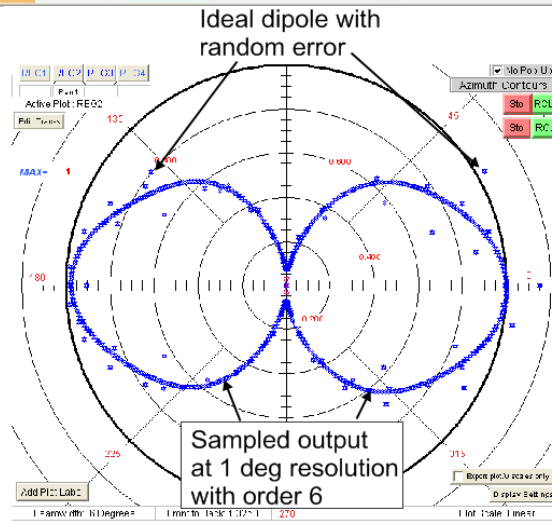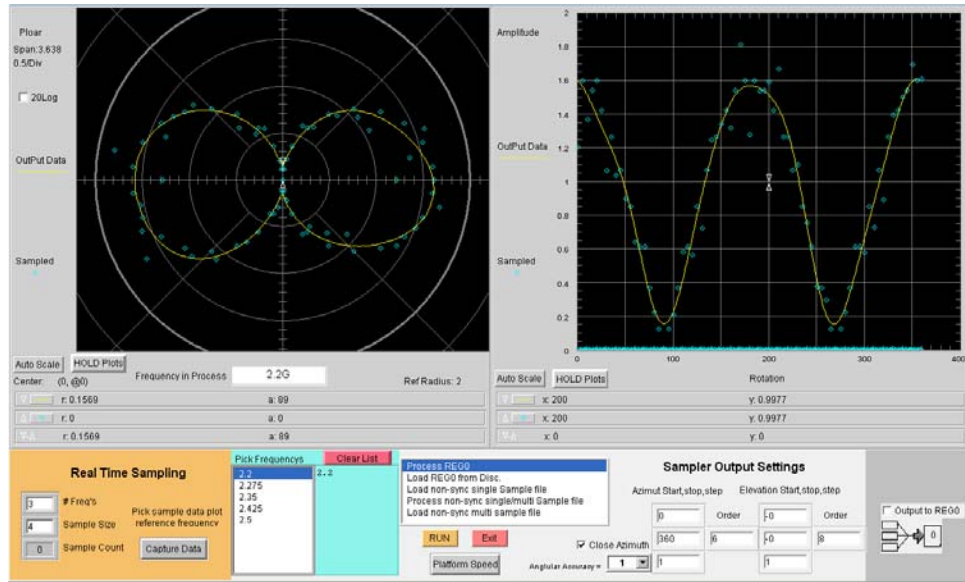
Next we ask how close to the exact original profile is the sampler output data? An overlay of both plots shows that the original data has been recovered to a high degree of accuracy.

Sampled error data output compared to original no error data
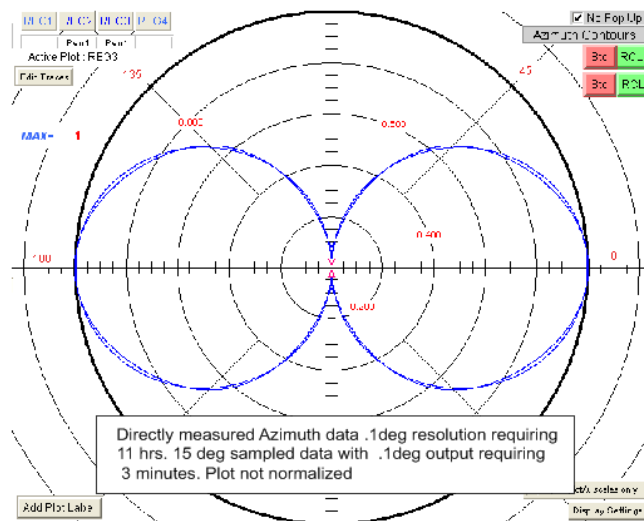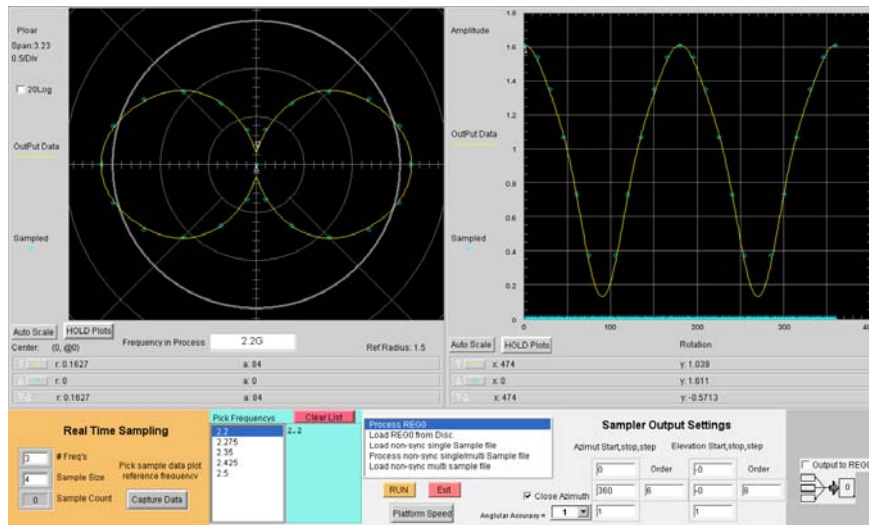
Sampler output
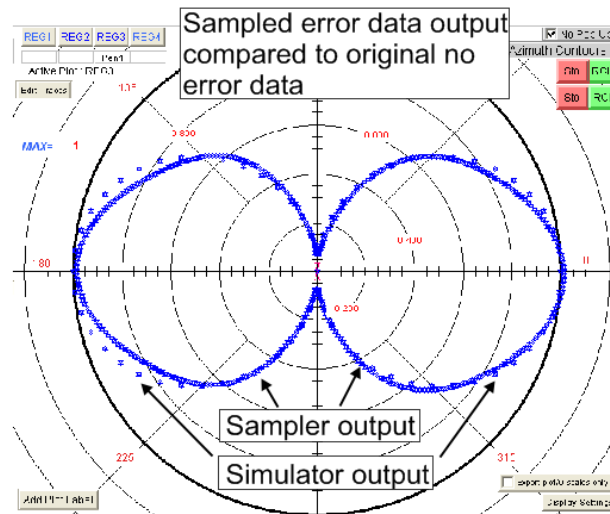
Simulator output

Example: Low resolution saving time and High resolution sampler output.

In this example we again use the dipole simulator and scale the measurement minutes to hours instead of minutes. Assume it necessary to measure a large heavy dipole to a resolution of .1 deg. If we measure directly at .1deg resolution the DAMs clock shows:

STATUS	0h:11m:15s Remaining
	Reading VNA

11 hours is required. Now since we have some knowledge of a dipole pattern we reduce the resolution to every 15 degrees and adopt a sampling order of 6 as before. This will require approximately 11hr*.1/15= 3.3 minutes. But is the data accurate?
The sampler plots below show an accuracy of the less than 1%.

Directly measured Azimuth data .1deg resolution requiring
11 hrs. 15 deg sampled data with .1deg output requiring
3 minutes. Plot not normalized

The 3 minute and 11 hour data are clearly indistinguishable.

1.7 Antenna Maximum Bit Rate

In digital communications it is well known each component in a system
may contribute to the overall bit error rate. These errors may originate form
a variety of sources but the most common is that of atmospheric noise. The
receiving antenna is subjected to noise generated by the temperature of the
local sky. The result is a signal to noise ratio developed at the antenna
output. This ratio results in a maximum possible bit error rate as
determined by Shannon's limit.

$$BitRate(\max) = BW\, Log_2(1 + S/N)$$

where   BW = The information bandwidth
S = Received signal power density integrated over the antenna frequency response $S21^2$.
N = Noise power determined by noise density integration

Shannon's limit yields a figure of merit by which modulation techniques may be evaluated. By plotting the antenna BR$_{max}$ verses distance a judgment can be made as to the maximum distance (in a given system) the antenna is useful.
The AUT *Pr* can be determined from Friis' equation

$$P_r = (\frac{\lambda}{4\pi d})^2 G_t G_r P_t$$

as

$$P_r = P_{TX} S_{21}^2 P_L G_T G_r$$

The variation in $S21^2$ may be due to the AUT gain variation and the AUT S22. The measurement S21 may be corrected for the AUT S22 as follows:

$$S21_{AUT}^2 = S21^2 (1 - S22_{AUT}^2)$$

This separates the AUT gain from the mismatch which may or may not be desired depending on whether or not the application will match the AUT.

The receive signal level S is calculated as the average *Pr* over *BW* or:

$$P_{av} = P_{tx} \frac{\int_{\omega 1}^{\omega 2} S21(\omega)^2 d\omega}{\omega 2 - \omega 1}$$

Where P_{tx} = Transmit power density magnitude.
This represents an equivalent power level extending over the entire information BW. A similar calculation for the received noise requires the S21 response be normalize to 1 at the minimum loss point.